The first chapter discusses some general concepts of analog computation. Chapter 2 briefly reviews the theory of ordinary differential equations. It brings out those properties which are important to computer users. Chapter 3, Programming of Differential Equations, describes how one draws a computer diagram (i.e., set-up diagram using block symbols of operational amplifier, potentiometer, multiplier, etc.) from the given differential equations, with emphasis on initial conditions, normalized equations, and adjoint equations. Chapter 4 describes the techniques of block diagram manipulation and of scaling, and shows how operational amplifiers are used in problem solving. Chapters 5 and 6 examine the problems of explicit and implicit function generations, respectively.

Chapter 7, Error-Reduction Techniques, treats several cases where the effects of component errors can be reduced. Chapter 8, Optimization Techniques: Gradient Methods for Finding Maxima and Minima, describes how the computer may be used to optimize solutions. It mainly presents a mathematical and heuristical approach of gradient optimization procedures, and will be of interest to control engineers and systems designers.

The last three chapters, together with most of the appendices, provide an extensive discussion of statistics and of computer implementation of statistical problems. Chapter 9, Estimation and Test of Hypotheses, discusses fundamental statistical concepts and shows how one may use statistical properties to solve engineering problems. Chapter 10, Experimental Design and Detection of Errors, considers how many data are necessary to meet the accuracy requirements of a statistical problem, and discusses techniques useful in detecting computer malfunctions. (These two chapters were contributed by Arnold Levine.) Chapter 11, Application of Statistics to Computer Operations, treats methods for simulating statistical problems on the computer.

Although this book does not discuss methods of solving problems, using finite-difference networks and using continuous field analogs, it does uniquely present methods of solving problems involving noise processes rather extensively, as well as some techniques on error-reduction and optimization. It is quite readable and well written. It should serve as an excellent text for a course on solving engineering problems by analog methods.

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45[K, X].—J. M. Hammersley & D. C. Handscomb, *Monte Carlo Methods*, John Wiley & Sons, Inc., New York; Methuen & Co., Ltd., London, 1964, vii + 178 pp., 19 cm. Price $4.75.

This book is an exceptionally clear and stimulating survey of applications of the Monte Carlo method. It is not a text; very few derivations are given. It is a guide to what has been done with the Monte Carlo method, to how the method should be applied, and to what should be done with this method in future research and applications.

The Monte Carlo method associates with a given problem a statistical problem to which the answer provides an answer to the original problem. The associated statistical problem is solved by a combination of analysis and computer simulation with pseudo-random numbers.

The book surveys applications to pure mathematics, to nuclear physics, to statistical mechanics, to mathematical statistics, and to theoretical chemistry. Many original results are presented. An original discussion is presented of percolation processes, which involve deterministic flows in random media.

Through many examples the book stresses that the most obvious formulation of a Monte Carlo problem is not always the best. The required answer is a statistical expected value. To attain a given accuracy with a specified probability, the number of times an unbiased statistic must be computed is proportional to the variance of the statistic. Different Monte Carlo formulations, all of which yield the same expected value, are shown to produce very different variances.

The chapters entitled "Conditional Monte Carlo" and "Percolation Processes" are particularly fascinating. In problems involving conditional probabilities, the proper embedding in a larger measure space may make the difference between practical solvability and unsolvability. In certain complex statistical problems, such as percolation processes, direct simulation is impractical. But these problems may be shown analytically to be equivalent to other statistical problems which are solvable by direct simulation with presently available computers.

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46[L].—A. R. Curtis, *Coulomb Wave Functions*, Royal Society Mathematical Tables, Volume 11, Cambridge University Press, New York, 1964, xxxv + 209 pp., 28 cm. Price $15.00.

Schrödinger's equation for a hydrogen-like atom or ion in a potential field takes the form

$$\frac{h^2}{2\mu}\frac{d^2R}{dr^2} + \left\{W + \frac{Ze^2}{r} - \frac{h^2}{2\mu}\frac{L(L+1)}{r^2}\right\} R = 0,$$

on separating in polar co-ordinates. Here $-e$ denotes the charge and W the total energy of the electron, and Ze denotes the nuclear charge. The most extensive existing tables of solutions of this differential equation are those of the National Bureau of Standards [1], [2]. In these tables the independent variable used is $(2\mu W)^{1/2} h^{-1}r$. In the present tables the independent variable is $x = \mu e^2 h^{-2} Zr$; with this choice, wave functions for different values of W can be compared directly for constant values of r, rather than for constant values of $rW^{1/2}$. Thus the standard form adopted for the differential equation is

$$\frac{d^2y}{dx^2} + \left\{a + \frac{2}{x} - \frac{L(L+1)}{x^2}\right\} y = 0,$$

in which x is positive, a $(= 2Wh^2Z^{-2}\mu^{-1}e^{-4})$ is real, and L is zero or a positive integer.

The tables cover the ranges $a = -2(0.2)2$; $L = 0, 1, 2$; $x = 0(0.1)10$ and $1/x = 0(0.002$ or $0.005)0.1$. The accuracy of the tabulated values is six decimals